1/14/20

No class on Thursday, Jan. 16

No class on Tuesday, Jan. 21

Given a sample of values $\{x_i\}_{i=1}^{n}$, then

$$s_n^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n} \text{ is an } \underline{\text{estimate}} \text{ of } \sigma_X^2$$

<u>Note</u>: $\sigma_X^2 = Var(X) = E[(X-\mu)^2]$

The corresponding $\underline{\underline{\text{estimator}}}$ for σ_X^2 is

$$\underline{\underline{S}}_n^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n} = \frac{\sum^{n}(X_i - \bar{X})^2}{n} \quad (\underline{\underline{\text{r.v.}}})$$

Consider $E[\underline{\underline{S}}_n^2]$

$$= \frac{1}{n} \cdot E[\Sigma (X_i - \bar{X})^2]$$

$$= \frac{1}{n} \cdot E[\Sigma (X_i^2 - 2\bar{X} X_i + \bar{X}^2)]$$

$$= \frac{1}{n} \cdot E[\Sigma X_i^2 - \Sigma 2\bar{X} X_i + \Sigma \bar{X}^2]$$

$$= \frac{1}{n} \cdot E[\Sigma X_i^2 - 2\bar{X} \underbrace{\Sigma X_i}_{n \cdot \bar{X}} + n \cdot \bar{X}^2]$$

$$= \frac{1}{n} \cdot E[\Sigma X_i^2 - 2n \cdot \bar{X}^2 + n \cdot \bar{X}^2]$$

$$= \frac{1}{n} \cdot E[\Sigma X_i^2 - n \cdot \bar{X}^2]$$

$$= \frac{1}{n} \cdot \left[E[\Sigma X_i^2] - E[n \cdot \bar{X}^2] \right]$$

$$= \frac{1}{n} \cdot \left[\sum E[X_i^2] - n \cdot E[\bar{X}^2] \right]$$

$$E[X_1^2] = E[X_2^2] = \cdots = E[X^2]$$

$$\Rightarrow \sum E[X_i^2] = n \cdot E[X^2]$$

$$= \frac{1}{n} \cdot \left[n \cdot E[X^2] - n\, E[\bar{X}^2] \right]$$

$$= E[X^2] - E[\bar{X}^2]$$

<u>Note</u>: $E[\bar{X}^2] = Var(\bar{X}) + (E[\bar{X}])^2$

$$= \frac{Var(X)}{n} + (E[X])^2$$

$$= E[X^2] - \left(\frac{Var(X)}{n} + (E[X])^2 \right)$$

$$= \underbrace{E[X^2] - (E[X])^2} - \frac{Var(X)}{n}$$

$$= Var(X) - \frac{Var(X)}{n} = \left(1 - \frac{1}{n}\right) \cdot Var(X)$$

$$\therefore \quad E[\underline{\underline{S}}_n^2] = \frac{n-1}{n} \cdot Var(X)$$

$\Rightarrow$ $\underline{\underline{S}}_n^2$ is a "biased" estimator of $\sigma_X^2 = Var(X)$

It is <u>asymptotically</u> <u>unbiased</u> since

$$\lim_{n \to \infty} E[\underline{\underline{S}}_n^2] = Var(X)$$

<u>Remark</u>: $\frac{n}{n-1} \cdot \underline{\underline{S}}_n^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \underline{\underline{S}}_{n-1}^2$ is an unbiased estimator of $Var(X) = \sigma_X^2$ since

$$E[\underline{\underline{S}}_{n-1}^2] = E\left[\frac{n}{n-1} \cdot \underline{\underline{S}}_n^2\right] = \frac{n}{n-1} \cdot E[\underline{\underline{S}}_n^2] = \frac{\cancel{n}}{\cancel{n-1}} \cdot \frac{\cancel{n-1}}{\cancel{n}} Var(X) = Var(X)$$

Module 1: Probability Models

Section 1: Severity Models

[M1S1: Severity Models]

Notation: X is the random variable representing (rvr)
the dollar amount of loss per accident
(per claim)
(per loss)

X - "severity random variable"
(sometimes called the ground-up loss r.v.)

$X > 0$ & unless told otherwise (or implied) it's continuous

Common Parameters we'll be interested in are:

1) Distribution Function (i.e. cumulative distribution function) cdf

$$F_X(t) = Pr(X \leq t)$$

2) Survival Function

$$S_X(t) = Pr(X > t)$$

Note: $S_X(t) = 1 - F_X(t)$

3) density function (i.e. ~~prob~~ probability density function) pdf

$$f_X(t) = \frac{d}{dt}[F_X(t)] = -\frac{d}{dt}[S_X(t)]$$

4) hazard function

$$h_x(t) = \frac{f_x(t)}{S_x(t)} \quad (\text{L-TAM's Force of Mortality})$$

Note: $h_x(t) = \frac{-\frac{d}{dt}[S_x(t)]}{S_x(t)} = -\frac{d}{dt}[\ln(S_x(t))]$

$$\Longrightarrow \quad S_x(t) = e^{-\int_0^t h_x(z)\,dz}$$

5) cumulative hazard function

$$H_x(t) = \int_0^t h_x(z)\,dz$$

$$\therefore \quad S_x(t) = e^{-H_x(t)}$$

$$\Longleftrightarrow \quad H_x(t) = -\ln(S_x(t))$$